CS 154 Introduction to Automata and Complexity Theory Handout 7		
Winter 1996	R. van Glabbeek	January 31, 1996

Problem Set 4 Date Due: February 7, 1996

Reading: Floyd and Beigel, Chapter 0-5 to be read by Wednesday February 7'th.

Homework: Do the following exercises (FB refers to Floyd and Beigel *The Language of Machines*). Each problem must be submitted on a separate sheet. The homeworks must be solved without any collaboration.

Office hours change Popular demand has prompted Nikolaj to move his office hours to Tuesday and Thursday 4-5 in Gates 193. Thus, the new office hours are one hour later than previously.

Remember: The midterm is Friday February 16'th at the Annenberg Auditorium in the Cummings Art Building. It will cover Chapters 1-6.

1. (15 points) Consider the following regular language L: $(a \cup b)^* a(a \cup b) \cup bb$.
   (a) Construct an NFA for L.
   (b) Determinize the NFA into a DFA.
   (c) Complement the DFA.
   (d) Extract a regular expression for the complemented DFA.
2. (15 points) Solve problem 4.7-15
3. (15 points) State whether the following languages are regular and prove your answer:
   (a) Given a fixed, arbitrary $v \in (a \cup b)^*$, let $L_v = \{wvw^R : w \in (a \cup b)^*, |w| > 0\}$.
   (b) $\{wvw^R : w, v \in (a \cup b)^*, |w| > 0\}$
   (c) $\{w$: the number of occurrences of 01 as a substring of w equals the number of occurrences of 10 as a substring of $w\}$

Extra credit problem (optional) One-dimensional checkers (due to Reynolds, appears in Manna, "Mathematical Theory of Computation"). Consider a one-dimensional (infinite) checkerboard where each square contains the letter a or b, for example,

$\cdots$	a	a	b	a	a	b	b	b	a	a	b	a	a	a	a	$\cdots$

There is always some word, starting and ending with b ($baabbbaab$ in the example above) with an infinite string of a's in both directions. A *move* in the game consists of jumping a b over a neighbor b to an a square as follows:

	Before move	After move
	$b\ \overset{-\!-\!\rightarrow}{b}\ a$	$a\ \ a\ \ b$
or	$a\ \overset{\leftarrow\!-\!-}{b}\ b$	$b\ \ a\ \ a$

Thus, when a b jumps it leaves a trail of two a's in the direction of the jump. An *initial board* consists of a finite word w (starting and ending with b). w is said to be a *winning word* if there is a finite sequence of moves that will lead to a *final board* consisting of one b (and the rest a's).

1. Express the set of all winning words as a regular expression over $\{a, b\}$.
   Hint: Consider the final board and play *backwards*.
2. Prove your result.