Extra credit We claim that the language generated by the regular expression R consists of all winning boards:

$$R = b \cup b(ba)^*(ab)^*b \cup b(ba)^*(ab \cup bbba)(bb)^+(ab)^*b \cup b(ab)^*(bb)^+(ba \cup abbb)(ba)^*b$$

We have to verify that each word in R is a winning board, and each winning word is in R. This may be taken as soundness and completeness of the construction.

Soundness: Each word in R is winning. We prove this by induction on the number of b's in w, where $w \in R$. For $k = 1$ there is only one word $b \in R$, and this is winning. By the induction hypothesis we assume that all words in R with k b's are winning. Let w be a word with $k+1$ b's. It has one of the following forms:

- bb, then b obtained by one jump is in R so bb is winning.
- $b(ba)^{i+1}(ab)^j b$, $i, j \geq 0$, then $b(ba)^i(ab)^j b \in R$ obtained by one leftmost jump is winning by the induction hypothesis.
- $b(ba)^i(ab)(bb)^{j+1}(ab)^l b$, where $i = l = 0$, then we obtain the word $bbaa(bb)^j \in b(ab)^*(bb) + (ba \cup abbb)(ba)^*b \subset R$ in one move.
- Similar arguments are used for $b(ba)^i(ab)^{j+1}b$, $b(ba)^{i+1}(ab \cup bbba)(bb)^{j+1}(ab)^l b$ et.c.

In all cases one can reduce w to a word v with k b's that is a member of R.

Completeness: We must show that each winning word w is a member of R. To this end we again use induction on the number of b's in w to show that $w \in R$. The base case is trivial $k = 1$: $b \in R$. For the inductive step assume that every winning word with k b's is in R, and let w be a winning word with $k+1$ b's. Hence in one step towards the winning configuration w reduces to a word v with k b's, where $v \in R$. We consider the possible cases for v:

- $v = b$, then $w = bb \in R$.
- $v = b(ba)^i(ab)^j b$, then
$$w \in \begin{pmatrix} & b(ba)^{i+1}(ab)^j b \\ \cup & b(ba)^i(ab)^{j+1}b \\ \cup & b(ba)^{i-1}bbba(ab)^{j-1}b \\ \cup & b(ba)^{i-1}abbb(ab)^{j-1}b \end{pmatrix} \subset R.$$
- $v = b(ba)^i ab(bb)^{j+1}(ab)^l b$, then
$$w \in \begin{pmatrix} & b(ba)^{i+1}ab(bb)^{j+1}(ab)^l b \\ \cup & b(ba)^i ab(bb)^{j+1}(ab)^{l+1}b \\ \cup & b(ba)^{i-1}ab(bb)^{j+2}(ab)^l b \\ \cup & b(ba)^{i-1}bbba(bb)^{j+1}(ab)^l b \end{pmatrix} \subset R$$
- The other cases are treated similarly.