CS 154 Introduction to Automata and Complexity Theory		Handout 6
Winter 1996	R. van Glabbeek	January 29

Solution Set 2

Problem 1

Graded by Stephen Wu and Kenneth Cheng.

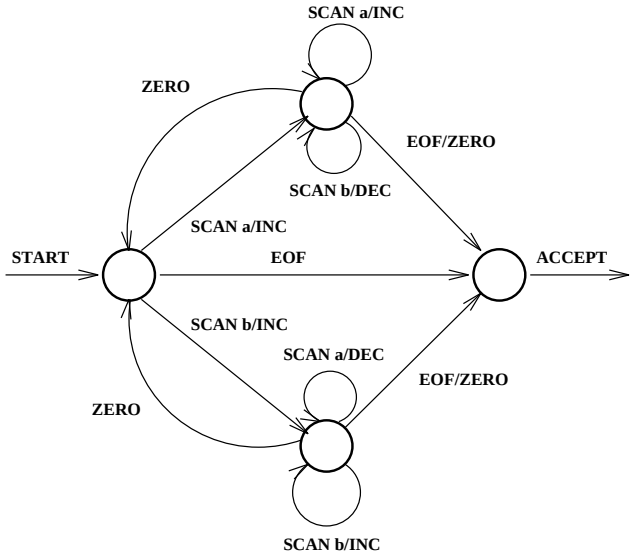

Figure 1: UCM accepting strings containing equally many a's and b's

Problem 2

Graded by Chung-Man Tam and Andy Hsieh

Assignment 2 , Problem 2 :

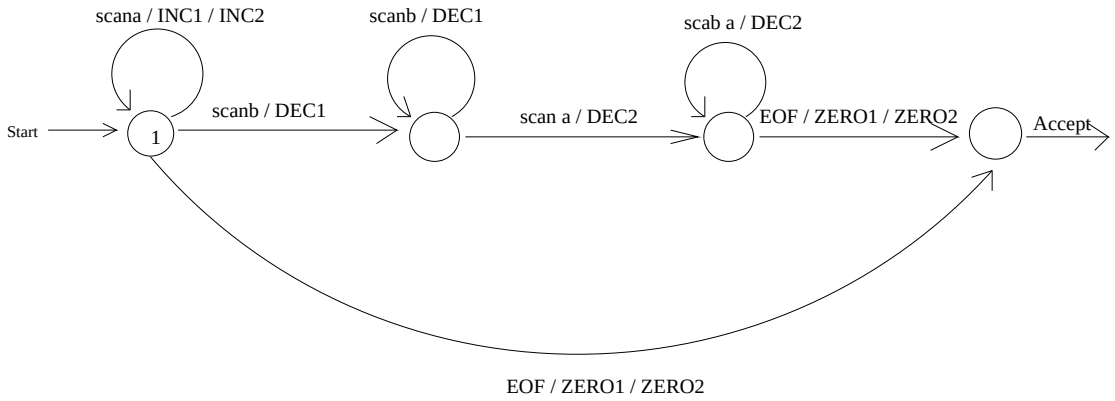

Figure 2: 2-UCM accepting $\{a^n b^n a^n \mid n \geq 0\}$

Extra credit problem

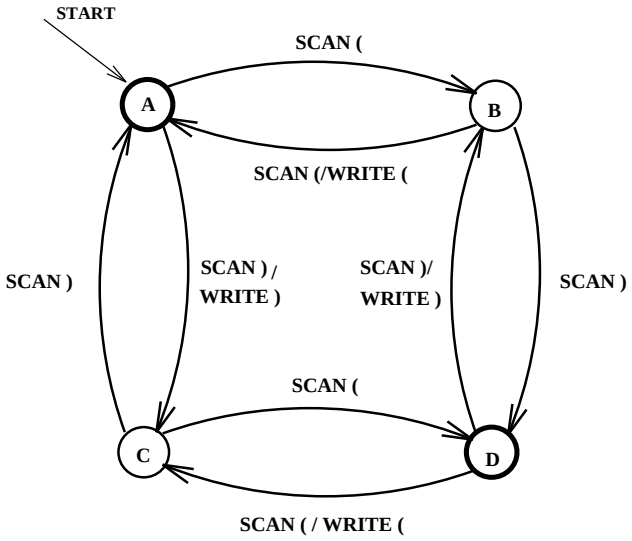

Figure 3: Finite parenthesis transducer

The finite transducer in Figure 3 solves the problem. Its initial state is labeled by A, and accepting states by A and D. To see that it indeed accepts and prints a balanced string of half the length of the original one must realize that the following invariant holds (we did not expect such an argument from those who solved the problem):

A When control resides at A, v is the input processed so far, and w is the output generated so far, then

$$(\exists m, n) \quad \left[\begin{array}{l} lp(v) = 2n \wedge \ rp(v) = 2m \wedge \\ lp(w) = n \ \wedge \ rp(w) = m \wedge \\ \text{balanced}(v \ \underbrace{)\ldots)}_{|2n-2m|} \ ) \iff \text{balanced}(w \underbrace{)\ldots)}_{|n-m|}) \end{array} \right]$$

B When control resides at B, v is the input processed so far, and w is the output generated so far, then

$$(\exists m, n) \quad \left[\begin{array}{l} lp(v) = 2n + 1 \wedge \ rp(v) = 2m \wedge \\ lp(w) = n \ \wedge \ rp(w) = m \wedge \\ \text{balanced}(v \ \underbrace{)\ldots)}_{|2n+1-2m|} \ ) \iff \text{balanced}(w \underbrace{)\ldots)}_{|n-m|}) \end{array} \right]$$

C When control resides at C, v is the input processed so far, and w is the output generated so far, then

$$(\exists m, n) \quad \left[\begin{array}{l} lp(v) = 2n \ \wedge \ rp(v) = 2m + 1 \wedge \\ lp(w) = n \ \wedge \ rp(w) = m \wedge \\ \text{balanced}(v \ \underbrace{)\ldots)}_{|2n-2m-1|} \ ) \iff \text{balanced}(w \underbrace{)\ldots)}_{|n-m|}) \end{array} \right]$$

D When control resides at D, v is the input processed so far, and w is the output generated so far, then

$$(\exists m, n) \quad \left[\begin{array}{l} lp(v) = 2n + 1 \wedge \ rp(v) = 2m + 1 \wedge \\ lp(w) = n \ \wedge \ rp(w) = m \wedge \\ \text{balanced}(v \ \underbrace{)\ldots)}_{|2n-2m|} \ ) \iff \text{balanced}(w \underbrace{)\ldots)}_{|n-m|}) \end{array} \right]$$

$lp(v)$ counts the number of left paranthesis in the string v, and $rp(v)$ counts the number of right parenthesis in v. The predicate balanced(v) holds if and only if v is a balanced string. $v\underbrace{)\ldots)}_{k}$ is the string obtained by concatenating k right parenthesis to the end of v.